CMP205: Computer Graphics

Lecture 5: Triangle Drawing and Hidden Surface Removal

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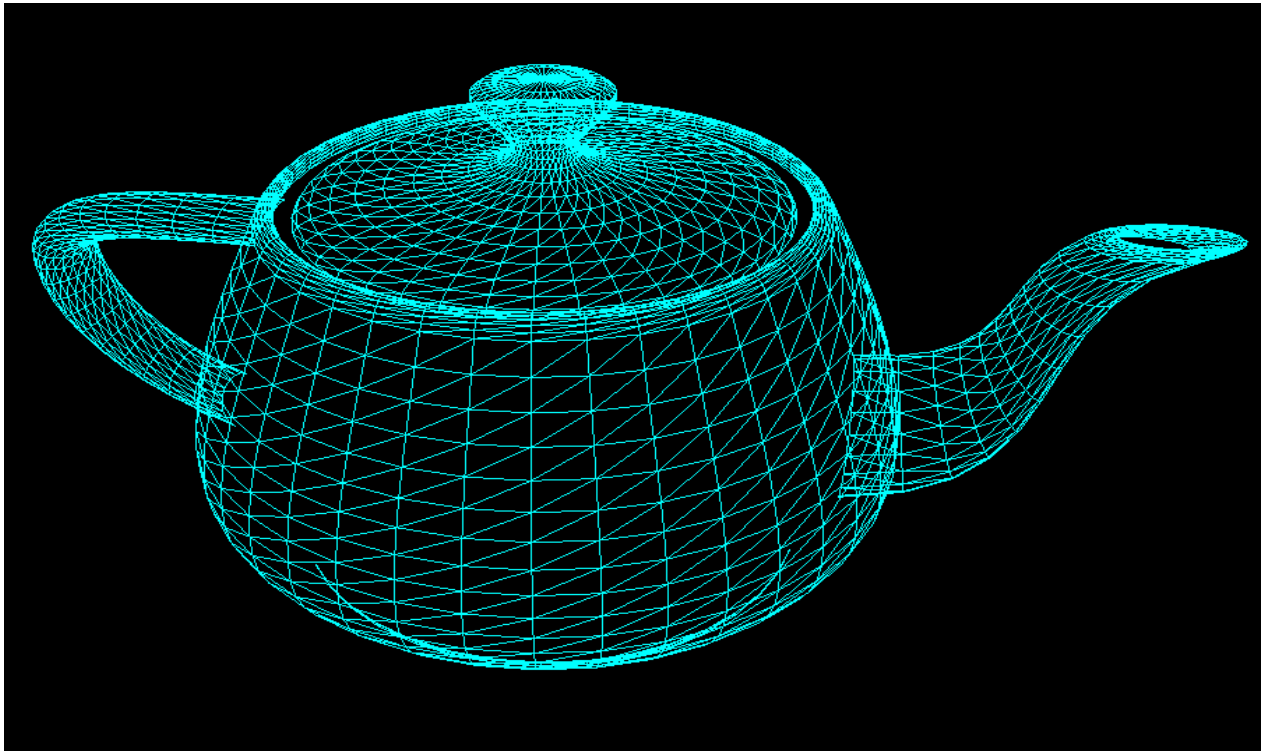
Agenda

- Triangle Drawing
- Anti-aliasing
- Z-Buffer

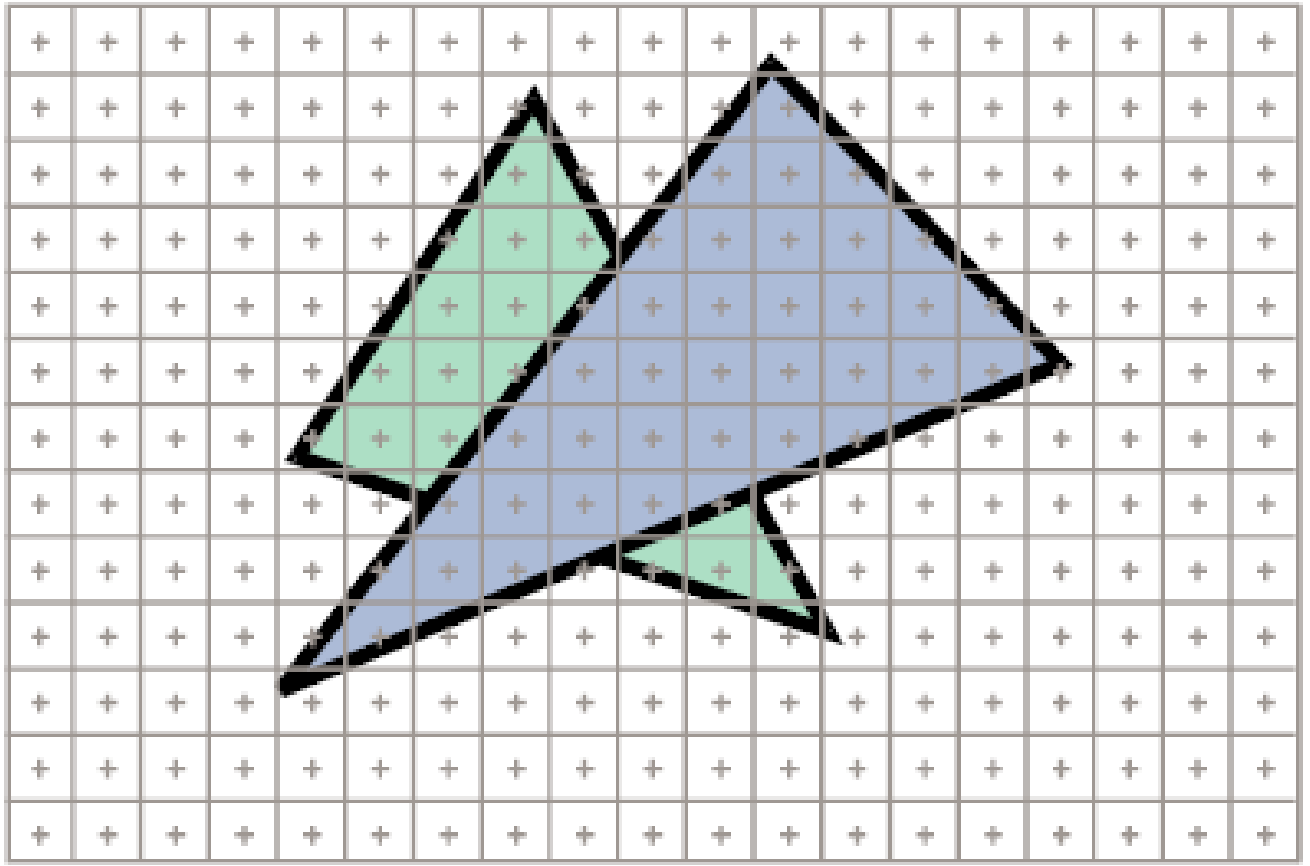
Acknowledgment: Some slides adapted from Steve Marschner and Maneesh Agrawala

Triangle Rasterization

- Second primitive shape, after the line!
- Used for modeling and shading surfaces
- Assign properties to vertices and interpolate

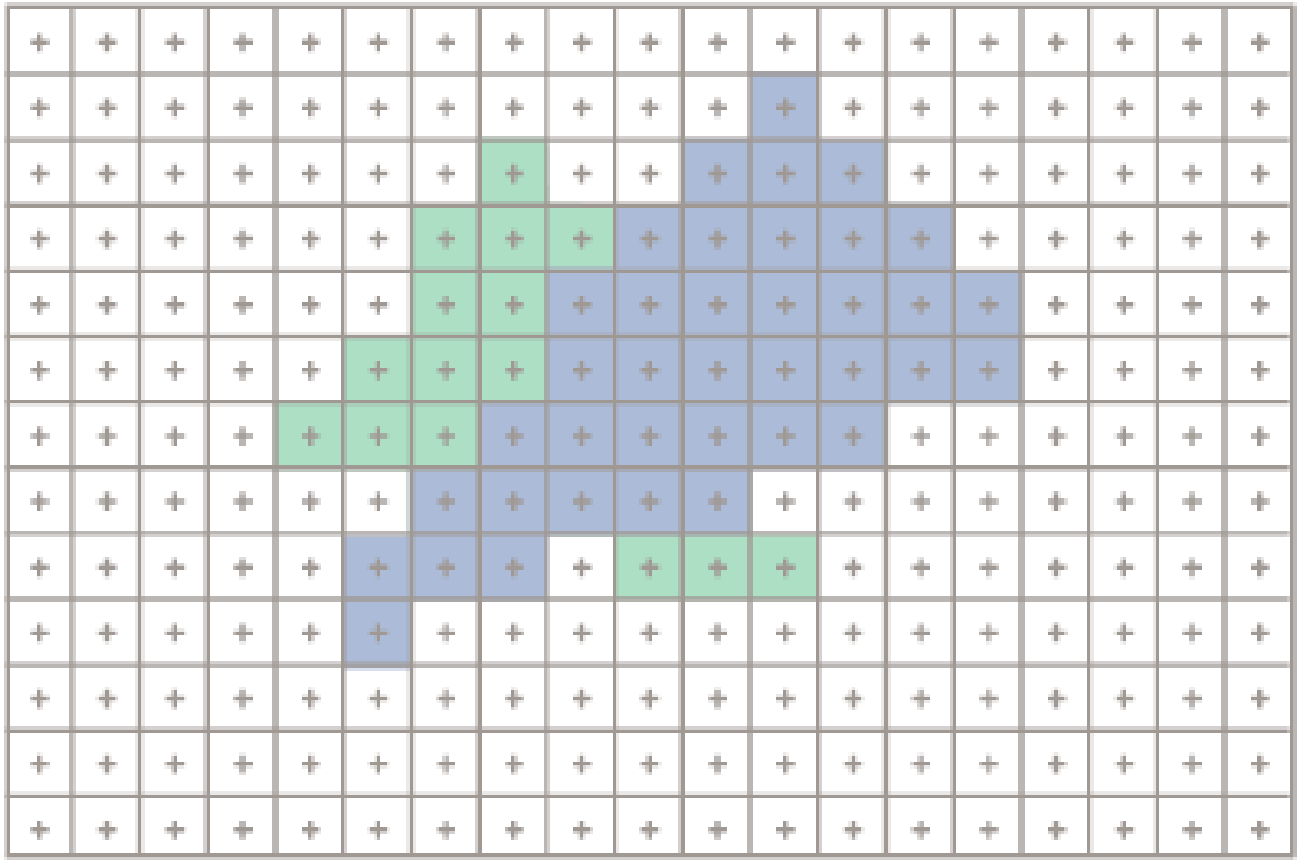


Triangle Rasterization



Given the three vertices of the triangle

Triangle Rasterization

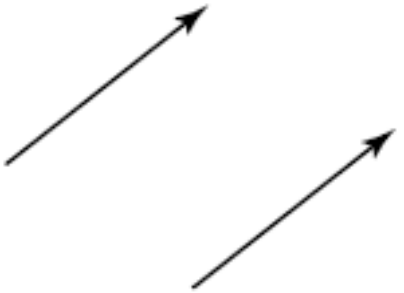


Find out which pixels belong to the triangle

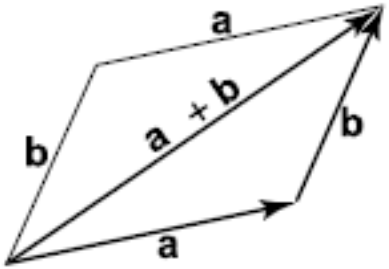
Vectors

A vector is a direction and a magnitude

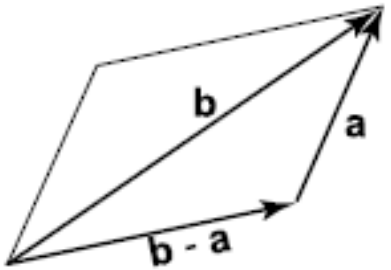
Two vectors are the same if they have the same direction and magnitude, even if they are at *different* places.



Vector Addition



Vector Subtraction



Vectors and Coordinates

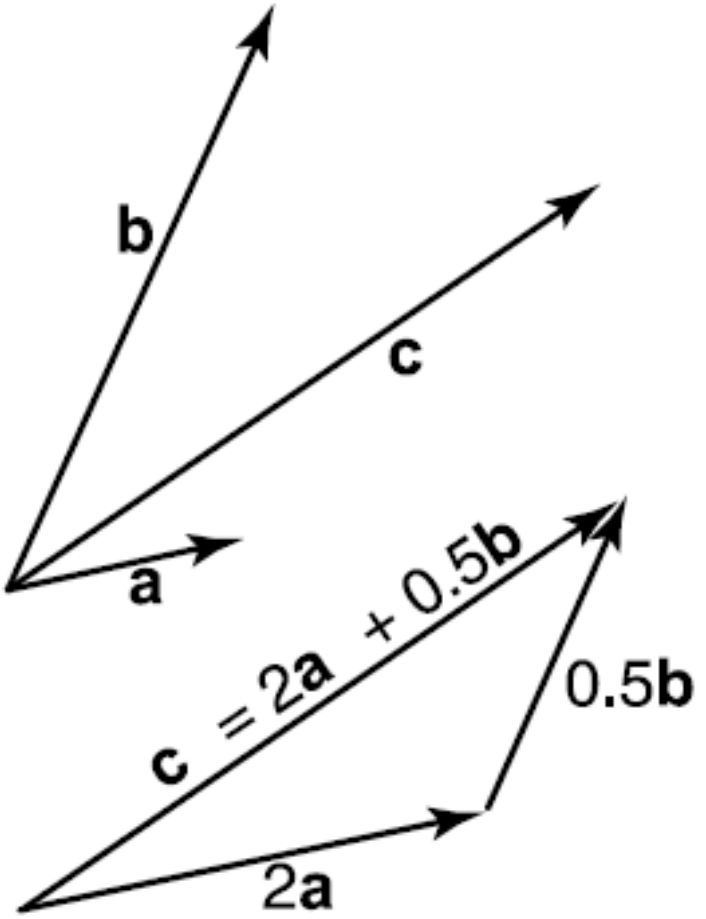
A 2D vector can be written as the combination of any two non-parallel vectors. This is called linear independence.

These two vectors are called the basis vectors.

$$c = c_a a + c_b b$$

We can then represent a vector as an ordered pair of numbers:

$$c = \begin{bmatrix} c_a \\ c_b \end{bmatrix}$$



Barycentric Coordinates

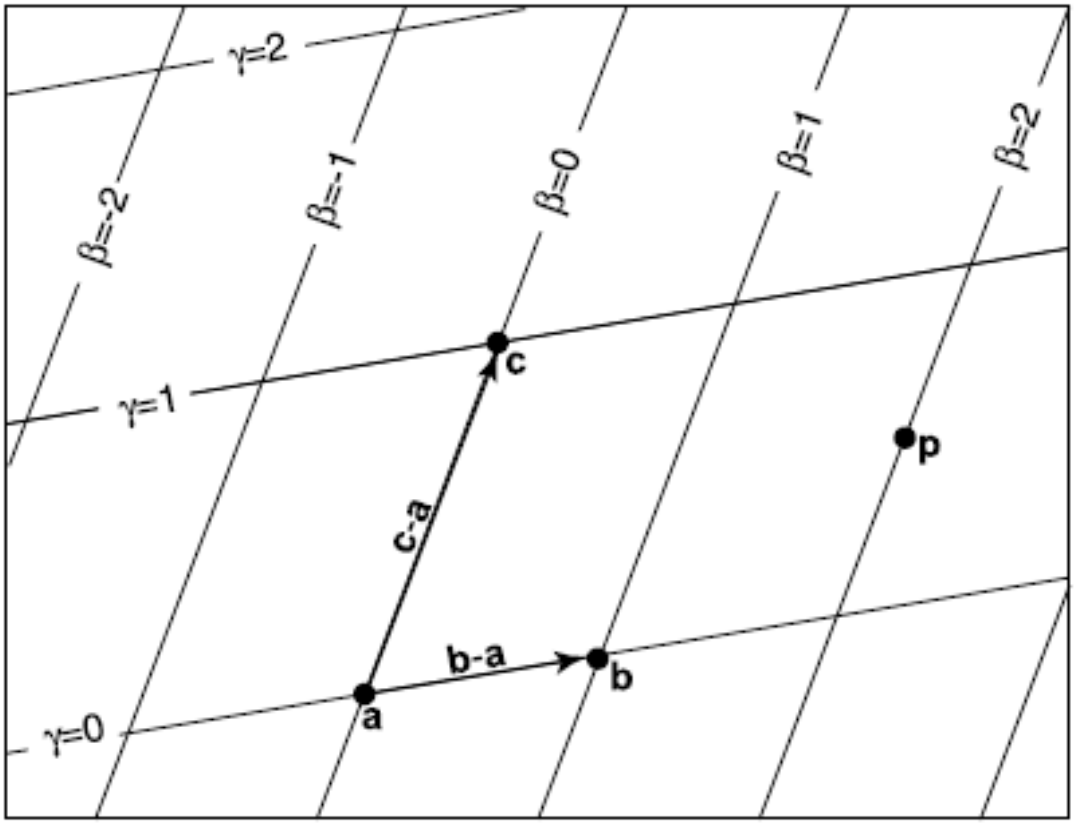
Any point p on the plane can be written as:

$$p = a + \beta(b-a) + \gamma(c-a)$$

$$p = (1-\beta-\gamma)a + \beta b + \gamma c$$

$$p(\alpha,\beta,\gamma) = \alpha a + \beta b + \gamma c$$

$$s.t. \quad \alpha + \beta + \gamma = 1$$



Color Interpolation:

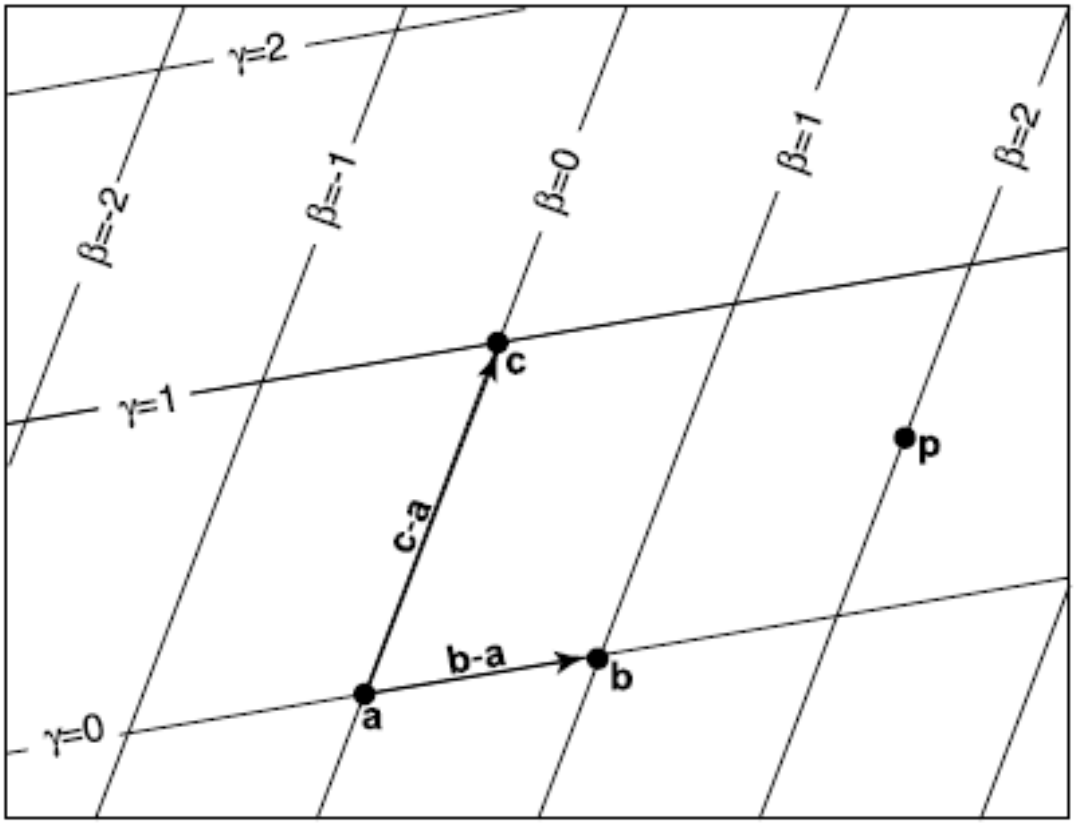
$$c_p = \alpha c_a + \beta c_b + \gamma c_c$$

Barycentric Coordinates

Inside Triangle iff:
$0 < \alpha < 1$
$0 < \beta < 1$
$0 < \gamma < 1$

On edge ab if:
$0 \leqslant \alpha \leqslant 1$
$0 \leqslant \beta \leqslant 1$
$\gamma = 0$

On vertex a if:
$\alpha = 1$
$\beta = 0$
$\gamma = 0$



$$p(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$

$$s.t. \quad \alpha + \beta + \gamma = 1$$

Computing Barycentric Coordinates

- Algebraic Solution
  - 2 equations in 2 unknowns

$$p = a + \beta(b-a) + \gamma(c-a)$$

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x_a \\ y_a \end{bmatrix} + \beta \begin{bmatrix} x_b - x_a \\ y_b - y_a \end{bmatrix} + \gamma \begin{bmatrix} x_c - x_a \\ y_c - y_a \end{bmatrix}$$

$$\begin{bmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} x_p - x_a \\ y_p - y_a \end{bmatrix}$$

Computing Barycentric Coordinates

- Geometric Solution

They are signed scaled distances from triangle sides:

Implicit Function: $\beta = f_{ac}(x, y)$

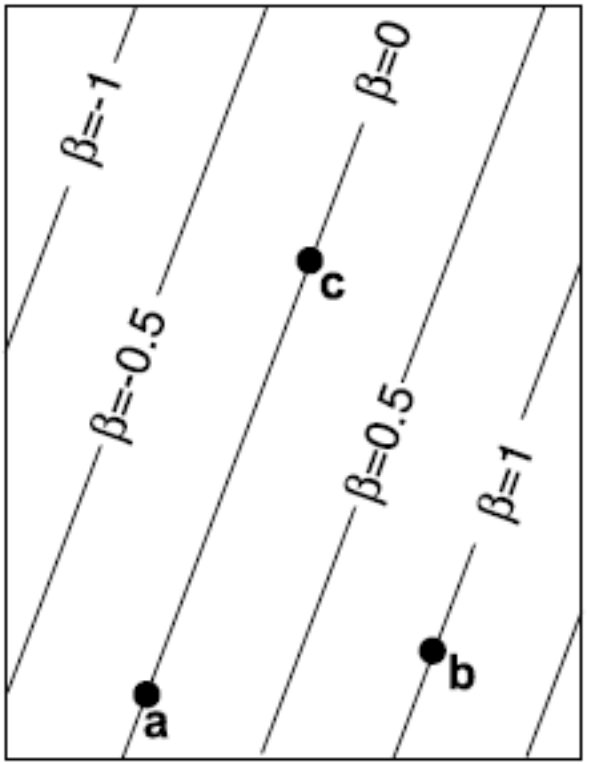
want:

$\beta = 0 \rightarrow$ on line ac

$\beta = 1 \rightarrow$ at point b

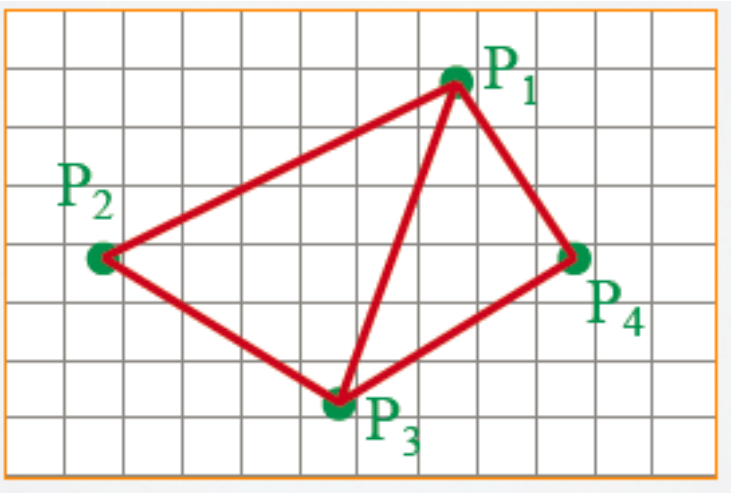
$$\beta = \frac{f_{ac}(x, y)}{f_{ac}(x_b, y_b)}$$

$$f_{ac}(x, y) = (y_a - y_c)x + (x_c - x_a)y + x_a y_c - x_c y_a$$



Triangle Rasterization

- We want it to be:
  - Fast
  - Accurate
  - No gaps
  - No order dependency



- Approach
  - Compute Barycentric Coordinates for every pixel
  - If inside triangle
    - Interpolate color
    - Draw

- What if pixels are *on* edges? Later.

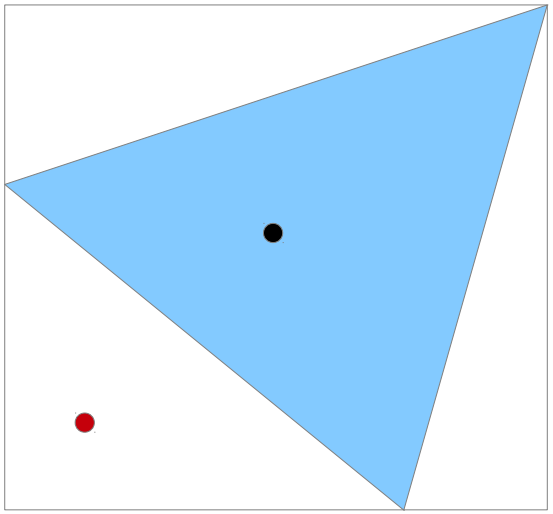
Triangle Rasterization

```
for all x do
  for all y do
    compute (alpha, beta, gamma) for (x,y)

    // Inside?
    if (alpha in (0,1) AND
        beta in (0,1) AND
        gamma in (0,1)) then
      c = alpha*c0 + beta*c1 + gamma*c2
      drawpixel(x,y) with color c
```

Optimizations?

Bounding Rectangles

Compute only for pixels likely to be inside the triangle



Every point inside the triangle *is* inside the bounding rectangle

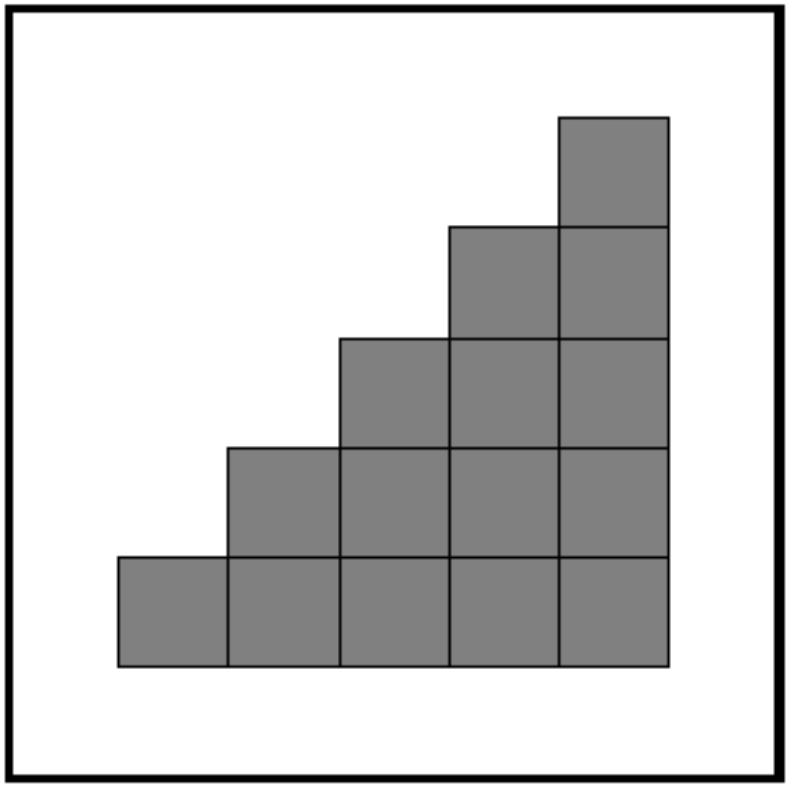
Not every point inside the bounding rectangle is inside the triangle

Triangle Rasterization

```
// Bounding rectangle
(xmin, xmax) = minmax(x0, x1, x2)
(ymin, ymax) = minmax(y0, y1, y2)

for x=xmin:xmax do
  for y=ymin:ymax do
    α = f12(x,y) / f12(x0,y0)
    β = f20(x,y) / f20(x1,y1)
    γ = f01(x,y) / f01(x2,y2)

    // Inside?
    if (α > 0 AND β>0 AND γ>0)
      c = α*c0 + β*c1 + γ*c2
      drawpixel(x,y) with color c
```

More Optimizations?

Triangle Rasterization

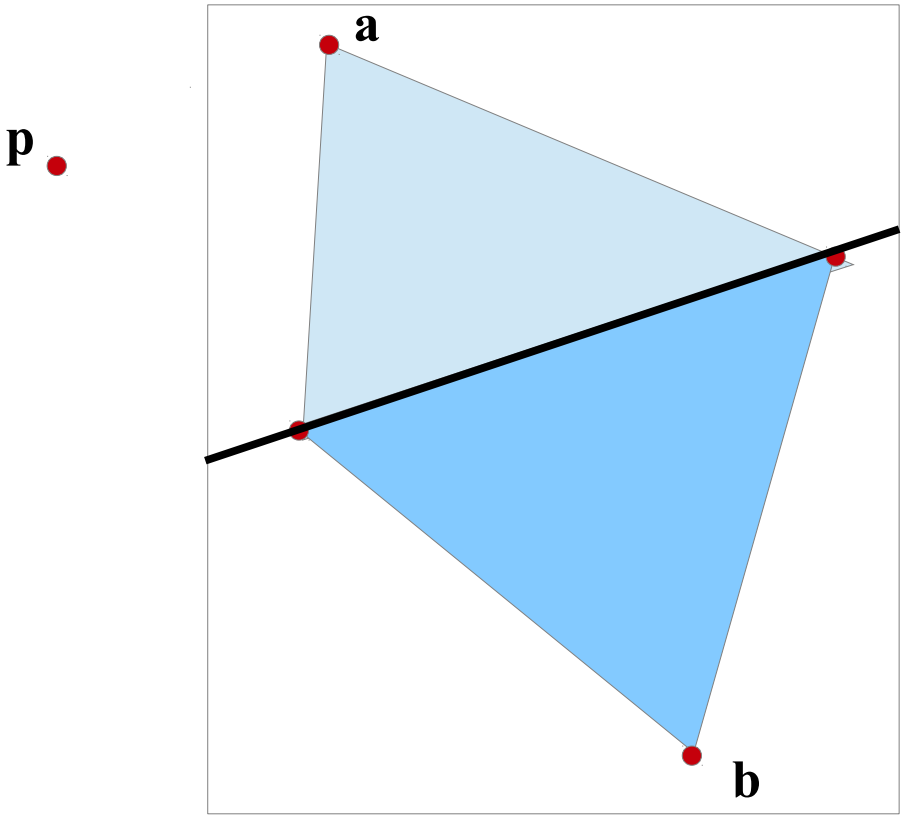


					0,00 1,00 0,00	
				0.25 0.75 0,00	0.25 1,00 0,00	
			0.50 0,50 0.00	0.50 0,75 0.00	0.50 1,00 0.00	
		0,75 0.25 0.00	0,75 0.50 0.00	0,75 0.75 0.00	0,75 1,00 0.00	
	1,00 0,00 0,00	1,00 0.25 0,00	1,00 0.50 0,00	1,00 0,75 0,00	1,00 1,00 0,00	

Triangle Rasterization

What to do with pixels on a shared edge? Which color should they get?

Pick a point p outside the screen e.g. (-1, -1)



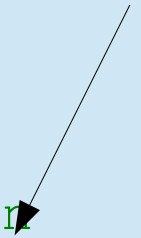
If the pixel on the shared edge belongs to the triangle whose opposite vertex is on the same side as p, then draw it

$$f(a) \times f(p) > 0 ?$$

Triangle Rasterization

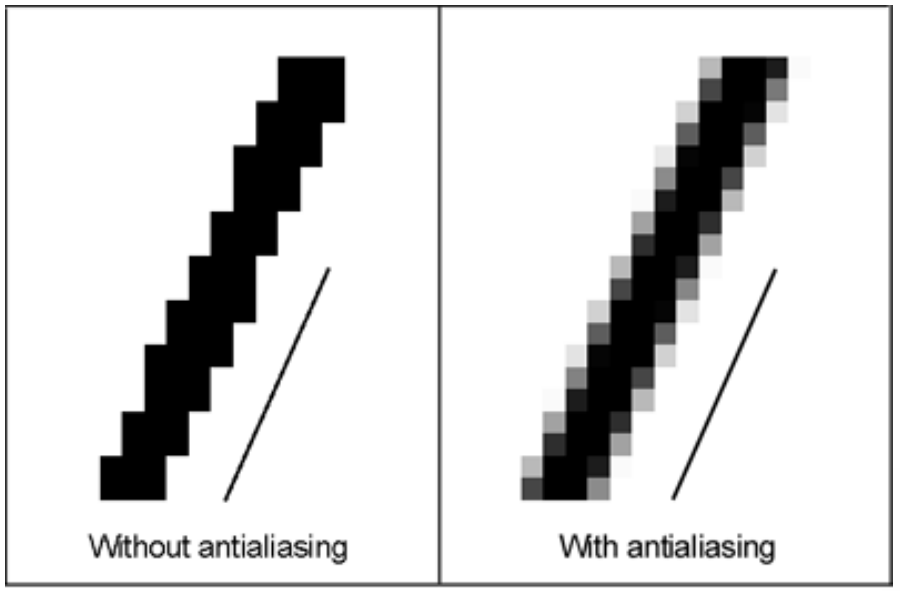
```
// Bounding rectangle
(xmin, xmax) = minmax(x0, x1, x2)
(ymin, ymax) = minmax(y0, y1, y2)

fα = f12(x0,y0)
fβ = f20(x1,y1)
fγ = f01(x2,y2)

for x=xmin:xmax do
  for y=ymin:ymax do
    α = f12(x,y) / fα
    β = f20(x,y) / fβ
    γ = f01(x,y) / fγ

    // Inside or on edge?
    if (α>=0 AND β>=0 AND γ>=0) then
      if (α>0 OR fα * f12(-1,-1) > 0) AND
         (β>0 OR fβ * f20(-1,-1) > 0) AND
         (γ>0 OR fγ * f01(-1,-1) > 0)) then
        c = α*c0 + β*c1 + γ*c2
        drawpixel(x,y) with color c
```

Inside the triangle or on the edge and belongs to the right triangle!

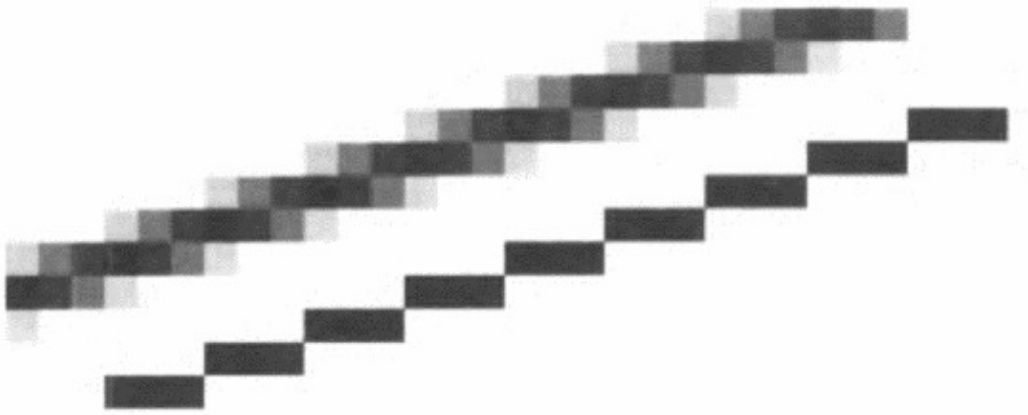


Anti-aliasing

Technique used to reduce the artifacts of sub-sampling and make the output more visually pleasing

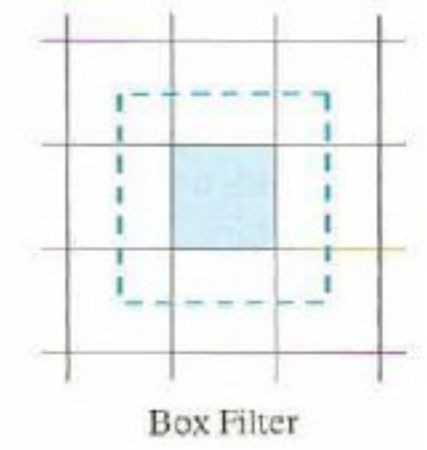


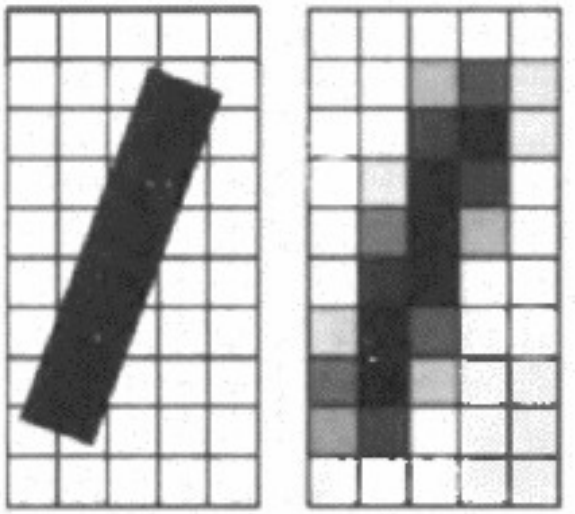
The transition from black to white is smooth



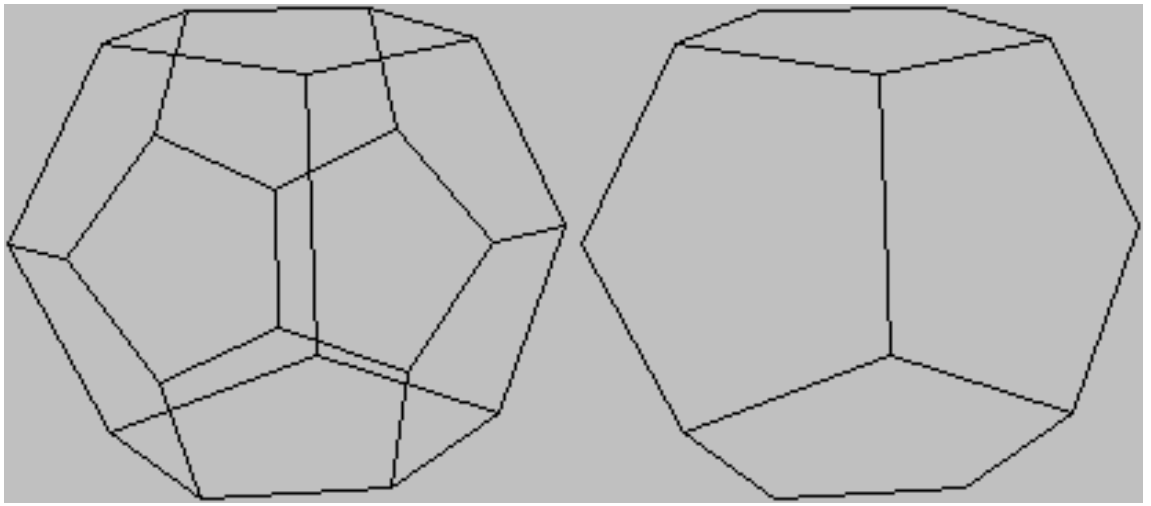
Antialiasing

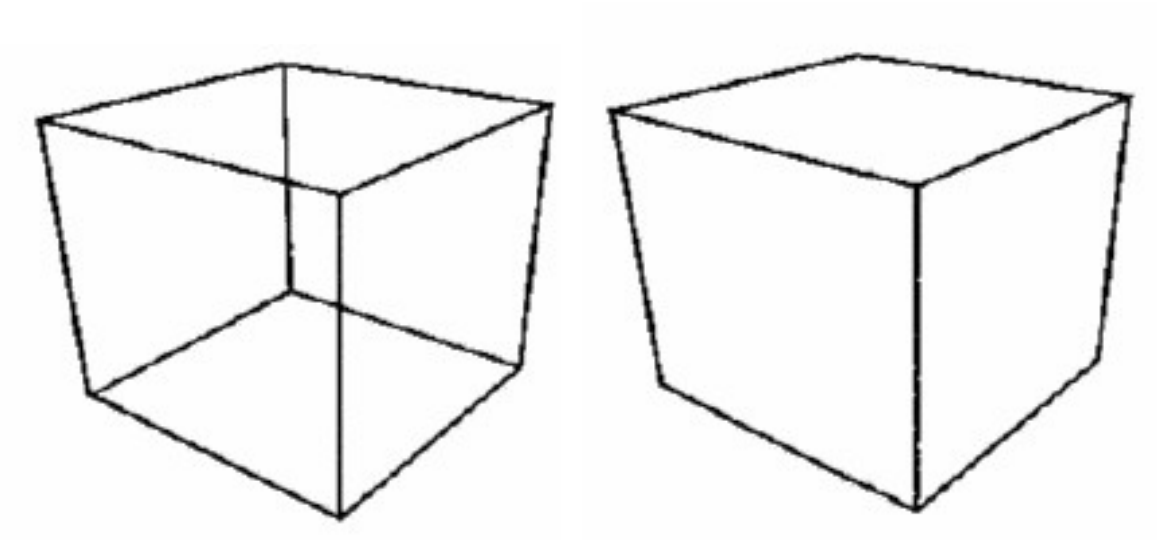
- Draw line at higher resolution
  - If we want a line on a raster of 256x256 pixels
  - Rasterize a line on a raster of 1024x1024
- "Box filter" to subsample
  - Replace every pixel with the average of its 16 neighbors





Hidden Surface Elimination

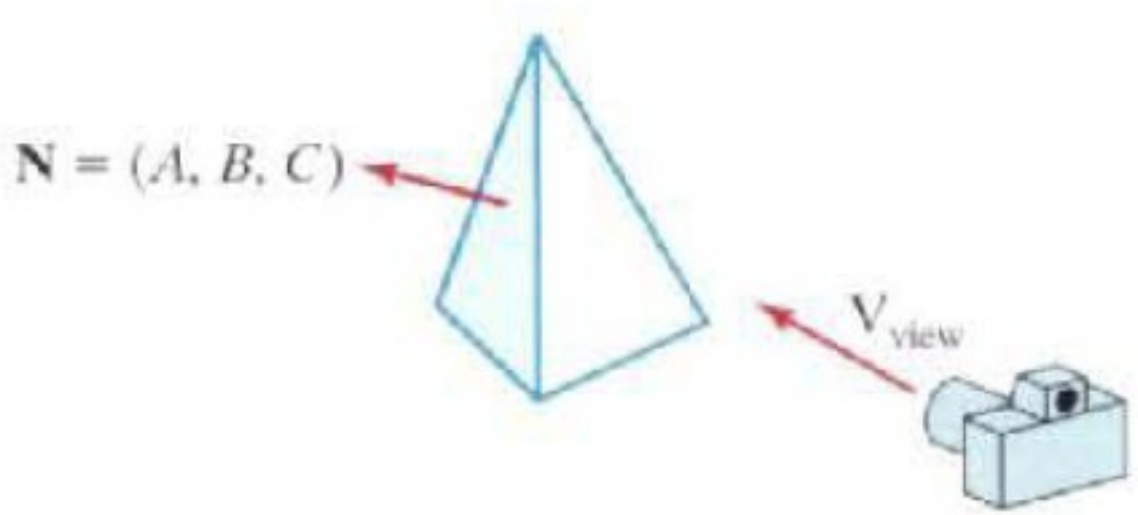




Back Face Elimination

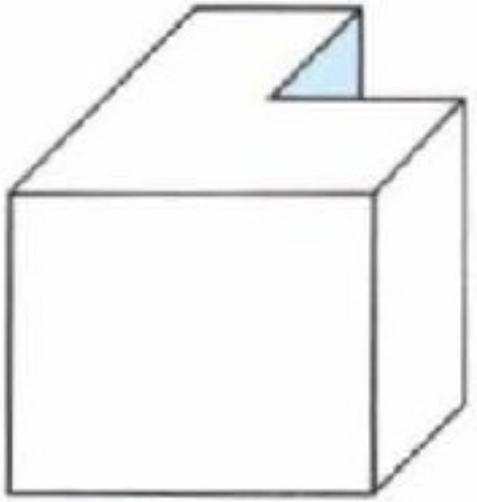
- Object Space
  - Works in the scene and compares objects
  - e.g. Back Face Culling
- Image Space
  - Works on the projected pixels
  - e.g. Z-Buffer and BSP Trees

Back Face Culling



If $N \cdot V_{view} > 0$ → Back Face

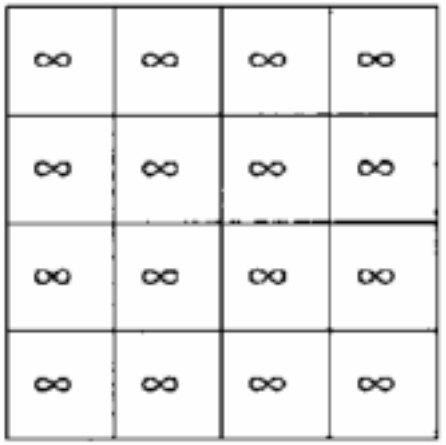
Is this enough?



Z-Buffer

$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = M_{vp} M_{orth} P M_{cam} M_m \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

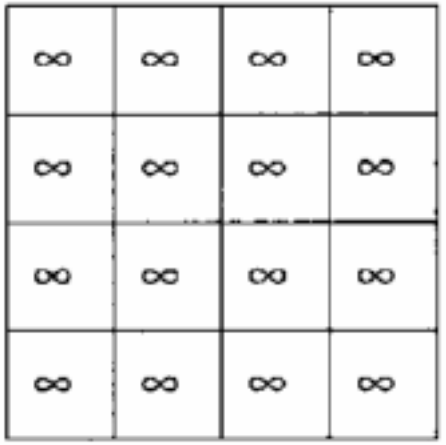
From the transformations pipeline, we get z_c



Keep a z value for every pixel on the screen !

Z-Buffer

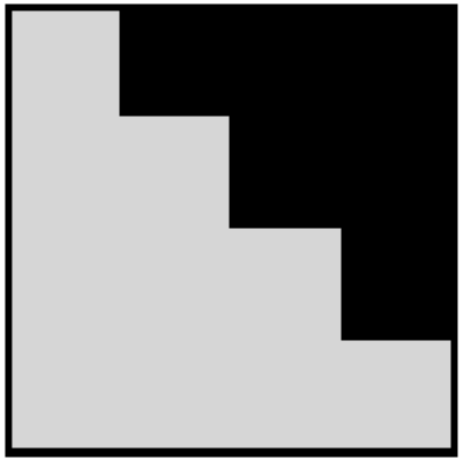
```
function SetPixel(i, j, color, z)
  if (z < z_buffer(i,j)) then
    z_buffer(i,j) = z
    screen(i,j) = color
```

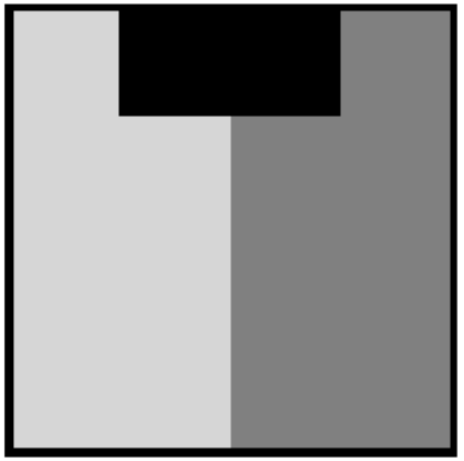


Z-Buffer

1	∞	∞	∞
1	3	∞	∞
1	3	5	∞
1	3	5	7

1	∞	∞	1
1	3	3	1
1	3	3	1
1	3	3	1

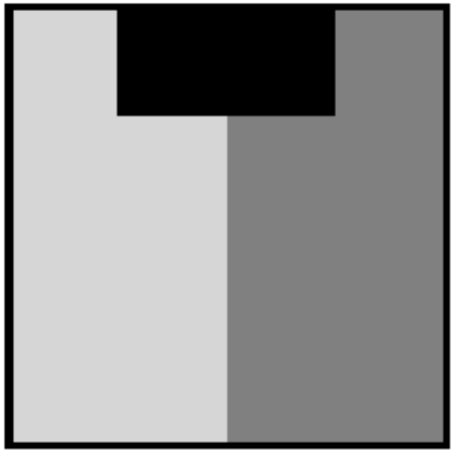




Left then right triangle

∞	∞	∞	1
∞	∞	3	1
∞	5	3	1
7	5	3	1

1	∞	∞	1
1	3	3	1
1	3	3	1
1	3	3	1

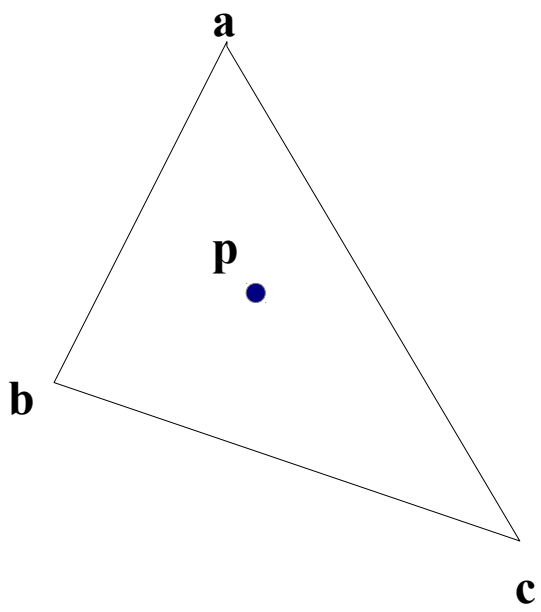


Right then left triangle

Z-Buffer independent of rendering order !

Z-Buffer

How do we get z values for pixels on a triangle ?!



Barycentric Coordinates !

$$p(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$

$$s.t. \quad \alpha + \beta + \gamma = 1$$

Recap

- Triangle Drawing
- Anti-aliasing
- Z-Buffer