CMP446: Computer Vision

CAIRO UNIVERSITY

Lecture 2: Camera Models and Projections

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Agenda

- Camera Model
- Geometric Primitives
- Transformations
- Projection
  - Orthographic
  - Perspective
- Camera Matrix

Some slides adapted from James Hays http://www.cs.brown.edu/courses/cs143/

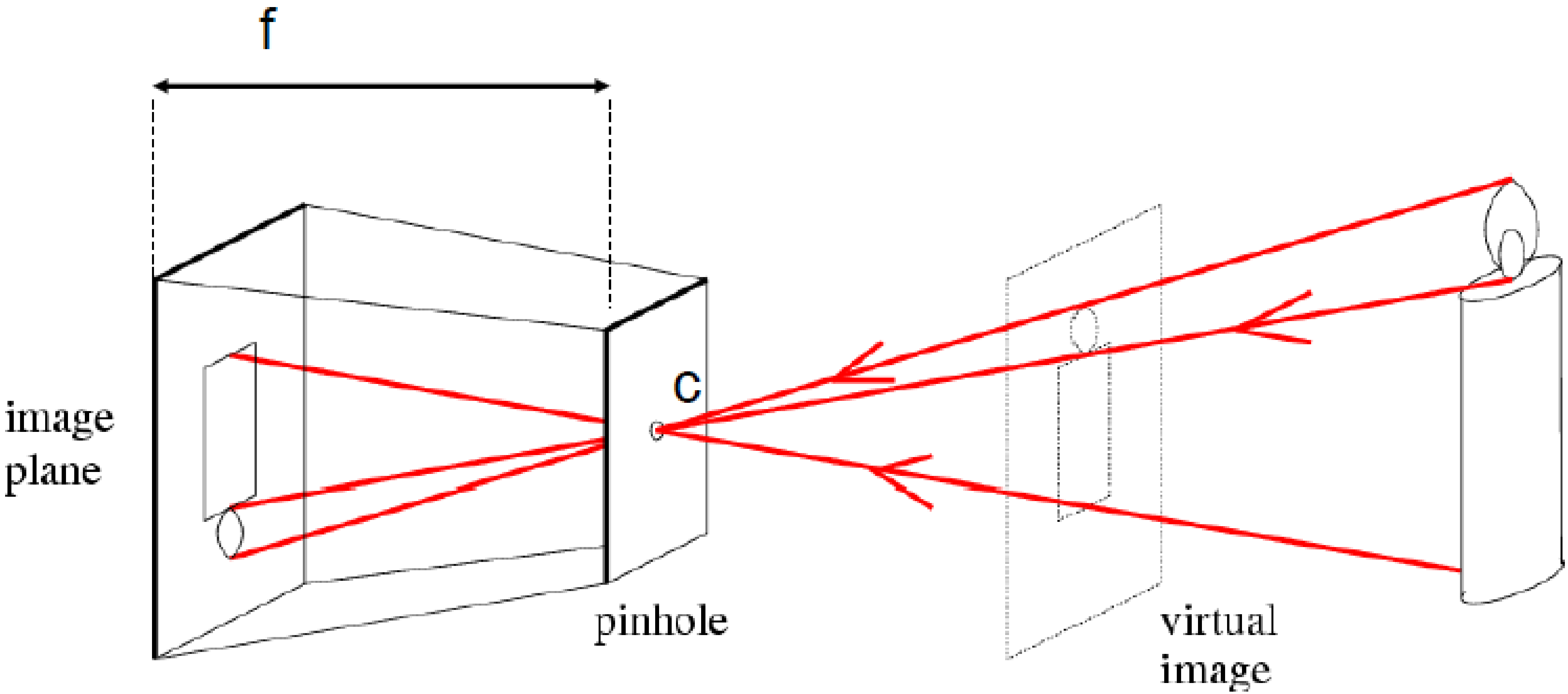
Goal

How do we go from 3D ...

Goal

… to 2D?

Pinhole Camera



f = focal length
c = center of the camera

Camera Obscura

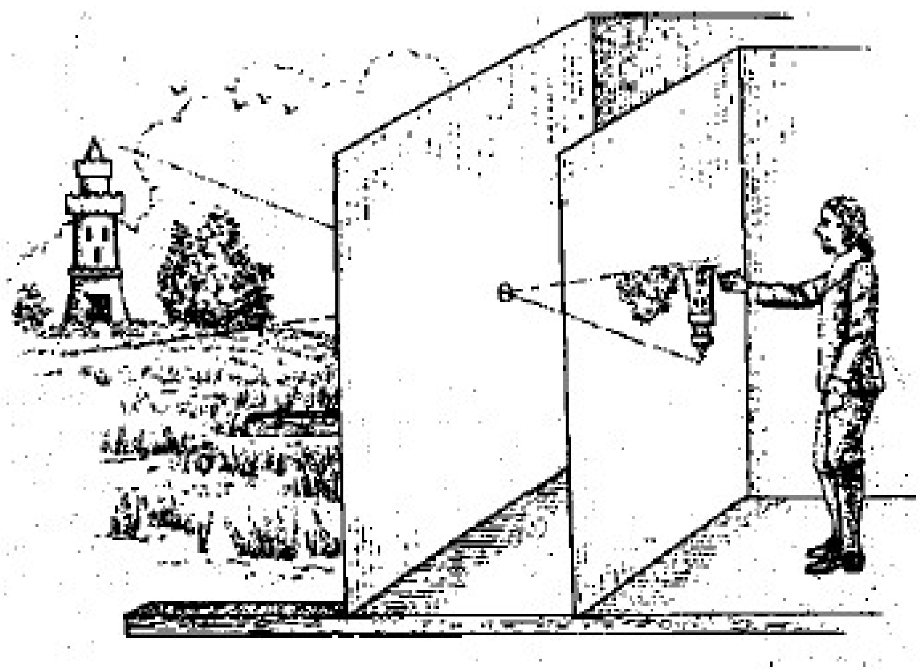


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Geometric Primitives: 2D

2D Point

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \in R^2$$

Augmented Vector

$$\bar{\boldsymbol{x}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix} = \tilde{w}\,\bar{x} \quad \in P^2$$

Geometric Primitives: 2D

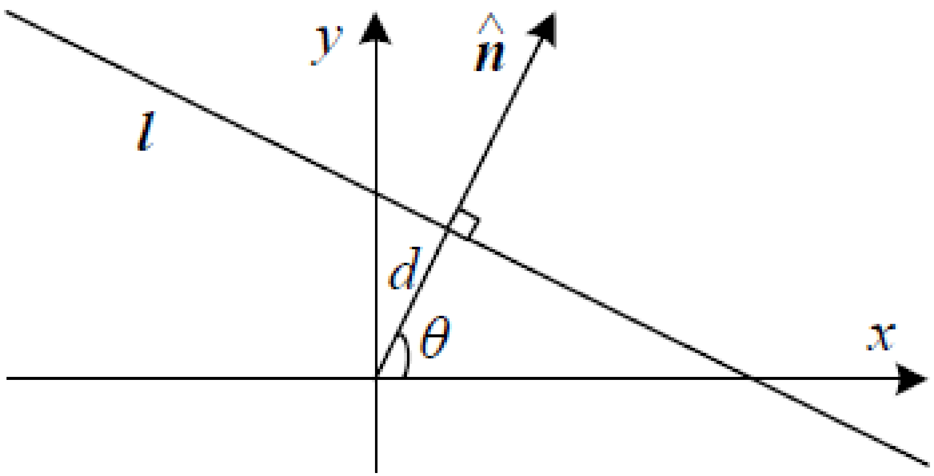
2D Line

$$\tilde{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \hat{n}_x \\ \hat{n}_y \\ d \end{bmatrix}$$

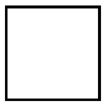
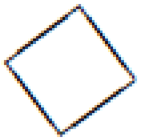
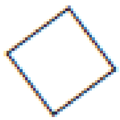
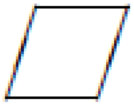
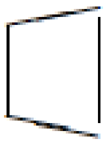
$$\tilde{l} \cdot \bar{x} = 0$$

Line at infinity

$$\tilde{l} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

Geometric Primitives: 3D

3D Point

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \in R^3$$

Augmented Vector

$$\bar{\boldsymbol{x}} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \tilde{w}\, \bar{x} \quad \in P^3$$

Geometric Primitives: 3D

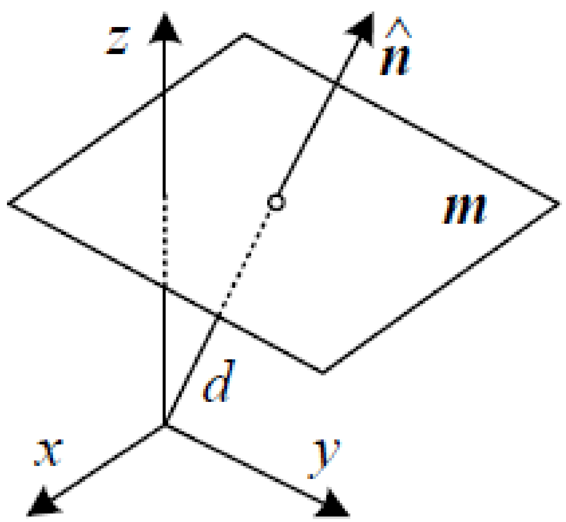
3D Line

$$\tilde{m} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \hat{n}_x \\ \hat{n}_y \\ \hat{n}_z \\ d \end{bmatrix}$$

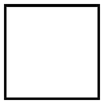
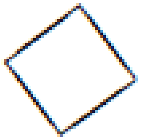
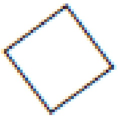
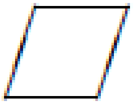
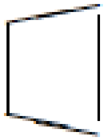
$$\tilde{l} \cdot \bar{x} = 0$$

Plane at infinity

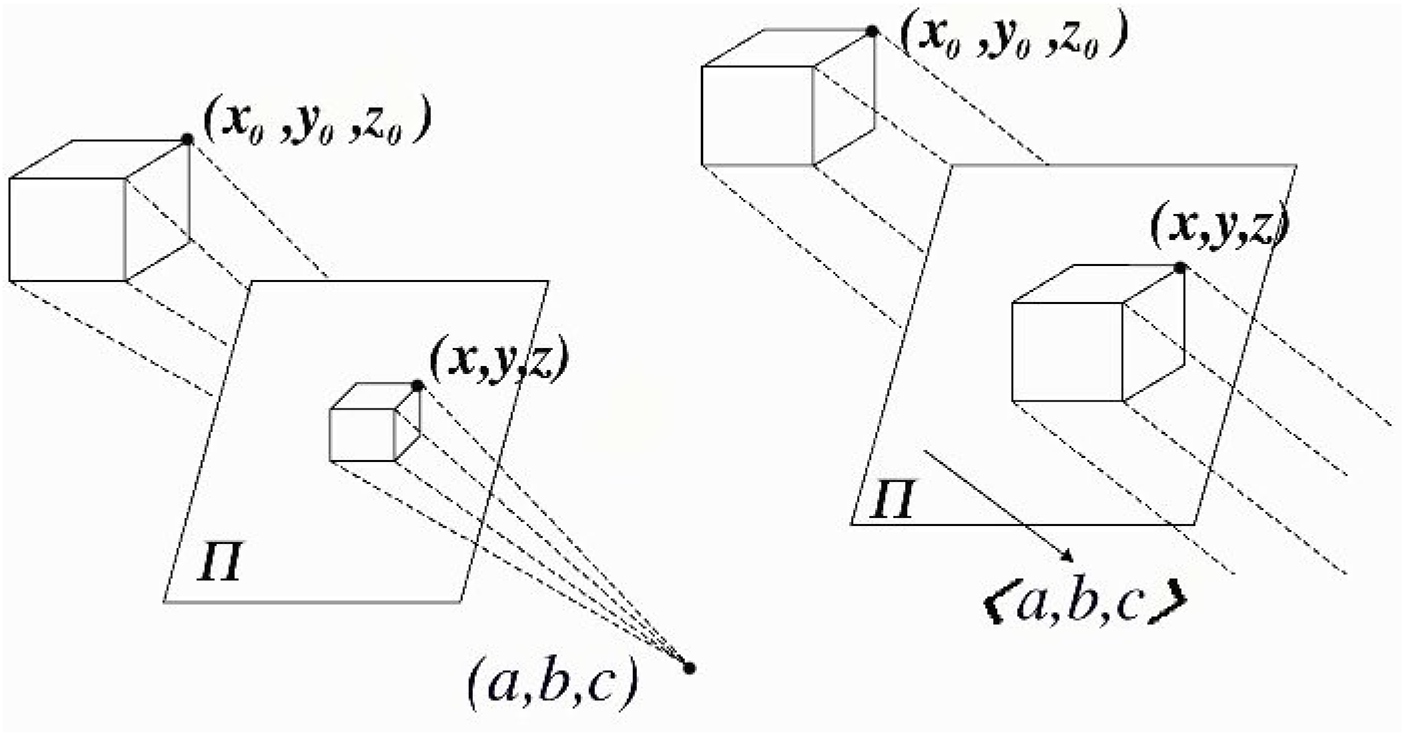
$$\tilde{l} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



3D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{3\times4}$	3	orientation	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{3\times4}$	6	lengths	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{3\times4}$	7	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{3\times4}$	12	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{4\times4}$	15	straight lines	

3D to 2D Projections

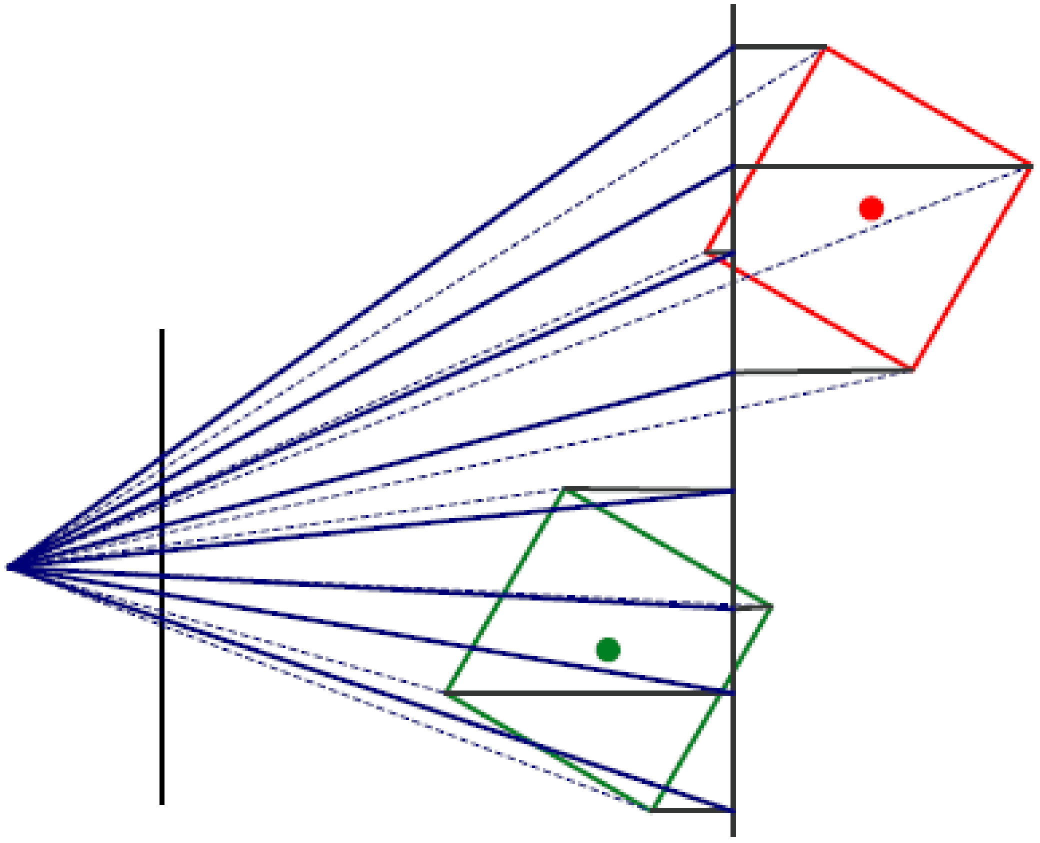


$$p \in R^3 \rightarrow x \in R^2$$
$$\tilde{p} \in P^3 \rightarrow \tilde{x} \in P^2$$

Orthographic Projection

$$\bar{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

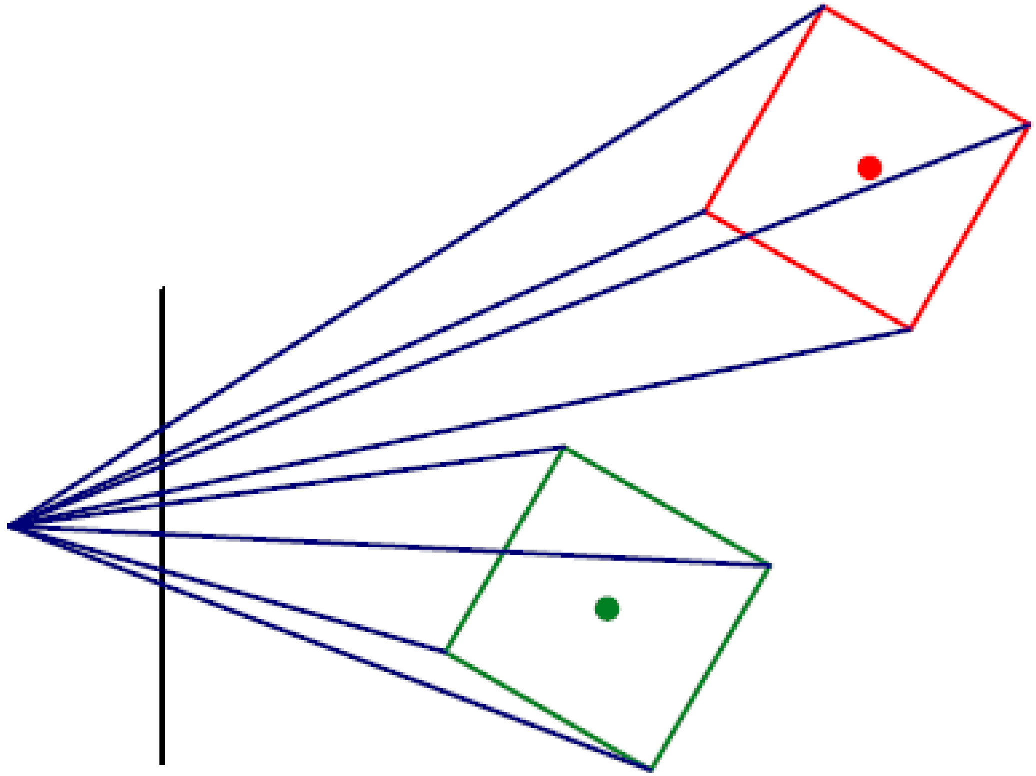
$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tilde{p}$$



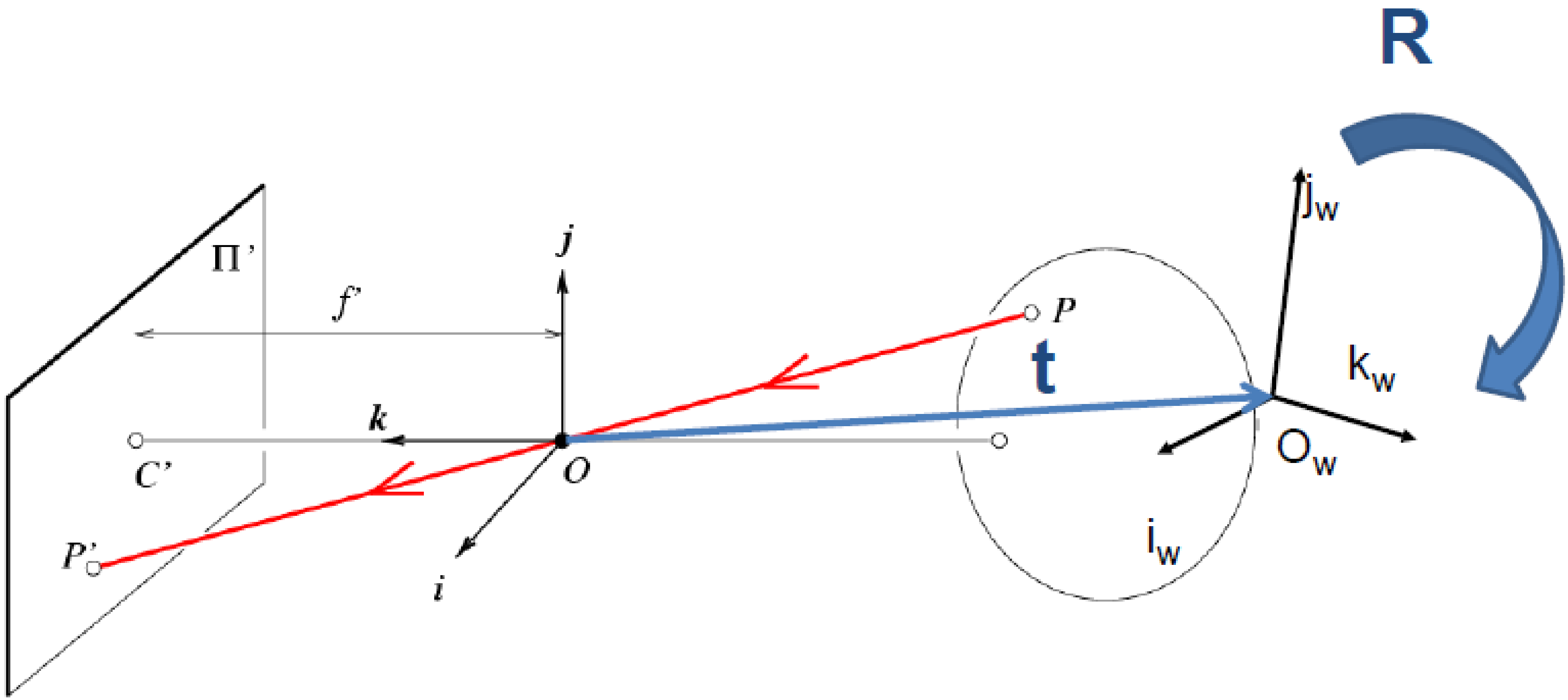
Perspective Projection

$$\bar{x} = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$

$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{p}$$

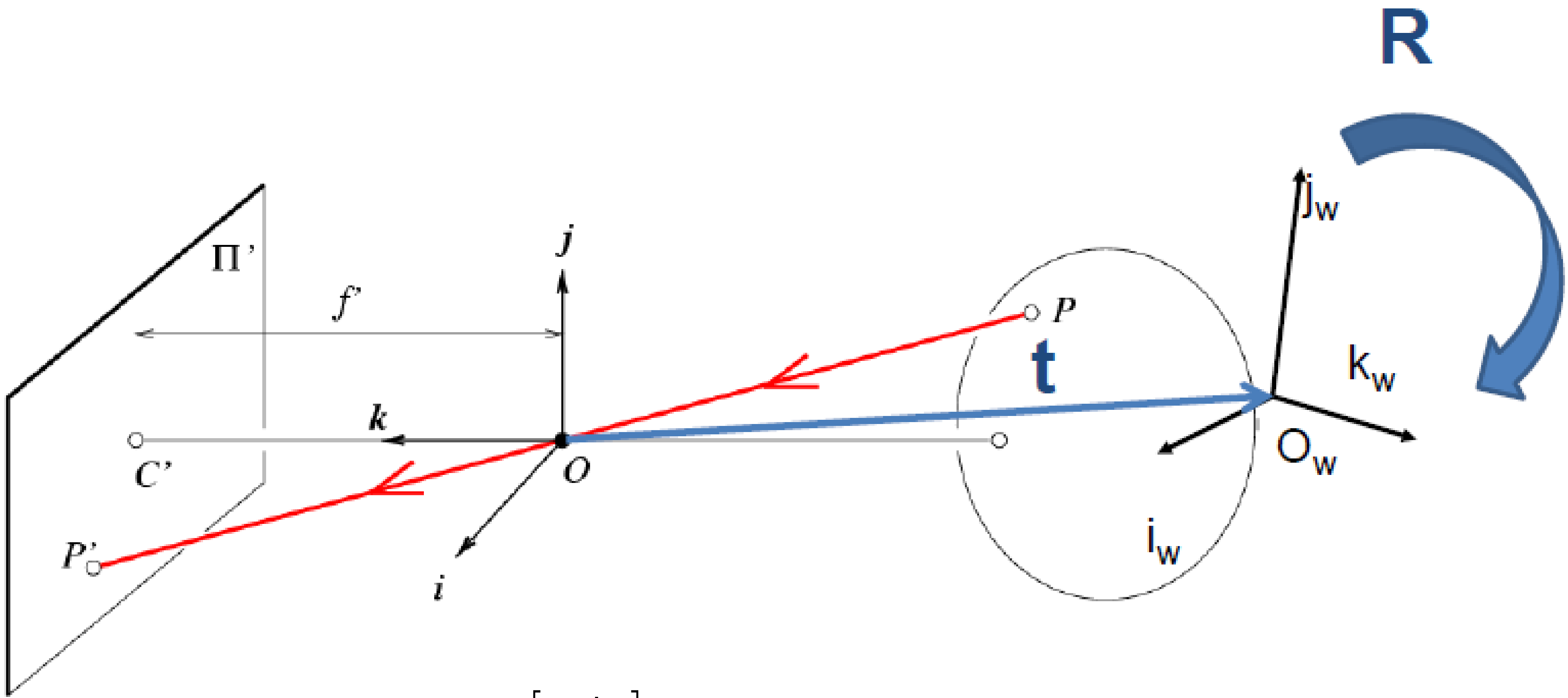


Camera Matrix



How to convert from world coordinates to image coordinates?

Camera Matrix



$$\tilde{x}_s = K\left[R|t\right]\tilde{p} = P\,\tilde{p}$$

K Intrinsic or Calibration Matrix

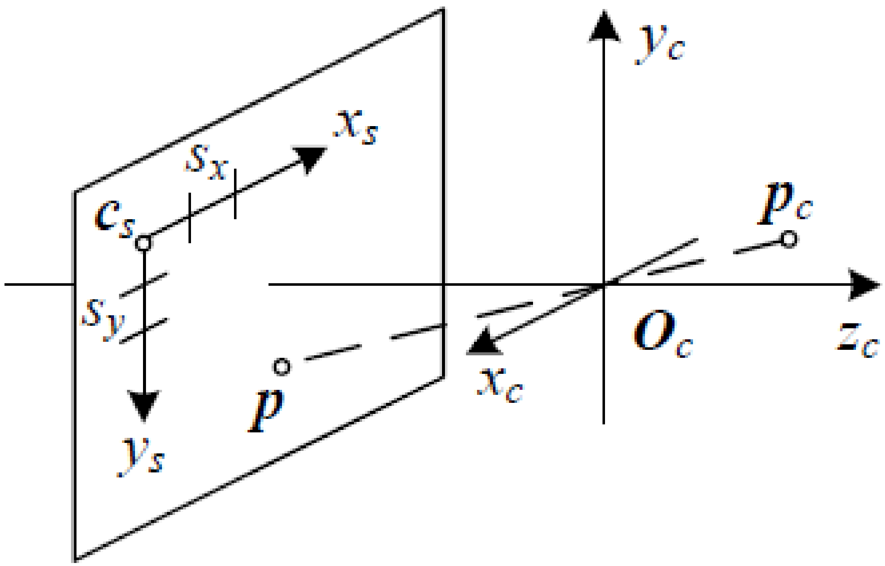
R and ***t*** are the Extrinsic Parameters

Camera Intrinsics

$$p_c = s\, p$$

$$p = \boldsymbol{M}_s \bar{\boldsymbol{x}}_s$$

$$p = \left[\boldsymbol{R}_s | \boldsymbol{c}_s \right] \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\tilde{\boldsymbol{x}}_s = \alpha\, \boldsymbol{M}_s^{-1} \boldsymbol{p}_c = \boldsymbol{K}\, \boldsymbol{p}_c$$

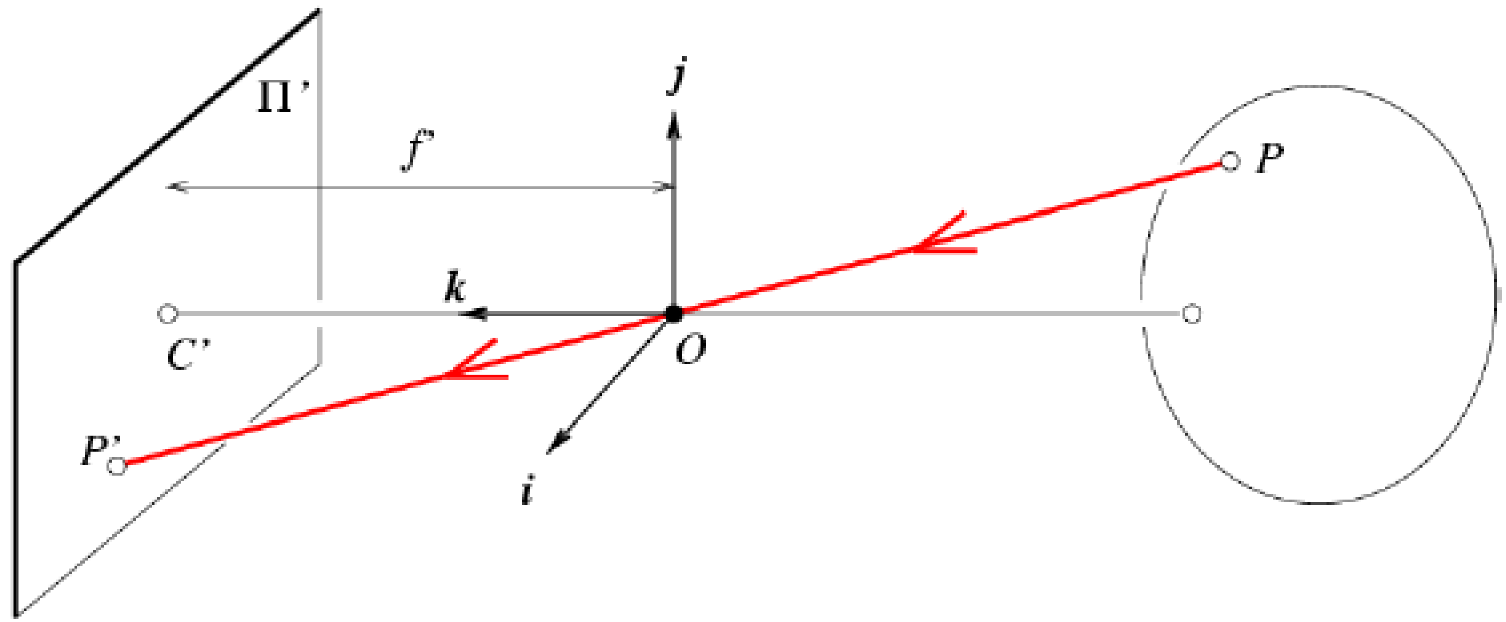
How many degrees of freedom?

Camera Intrinsics

$$\tilde{\boldsymbol{x}}_s = \overbrace{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\boldsymbol{K}} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

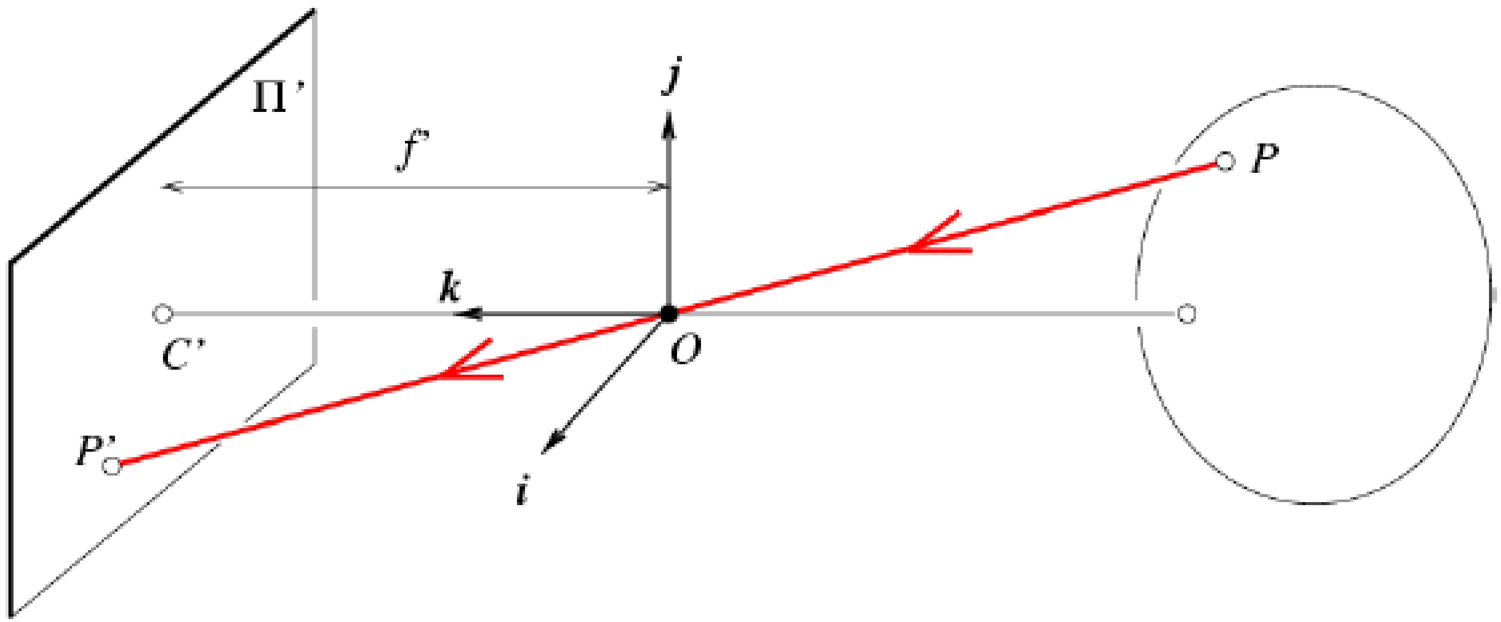
- Optical center at origin
- No skew
- Unit aspect ratio



Camera Intrinsics

$$\tilde{\boldsymbol{x}}_s = \overbrace{\begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\boldsymbol{K}} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:
- Optical center at origin
- No skew
- ~~Unit aspect ratio~~

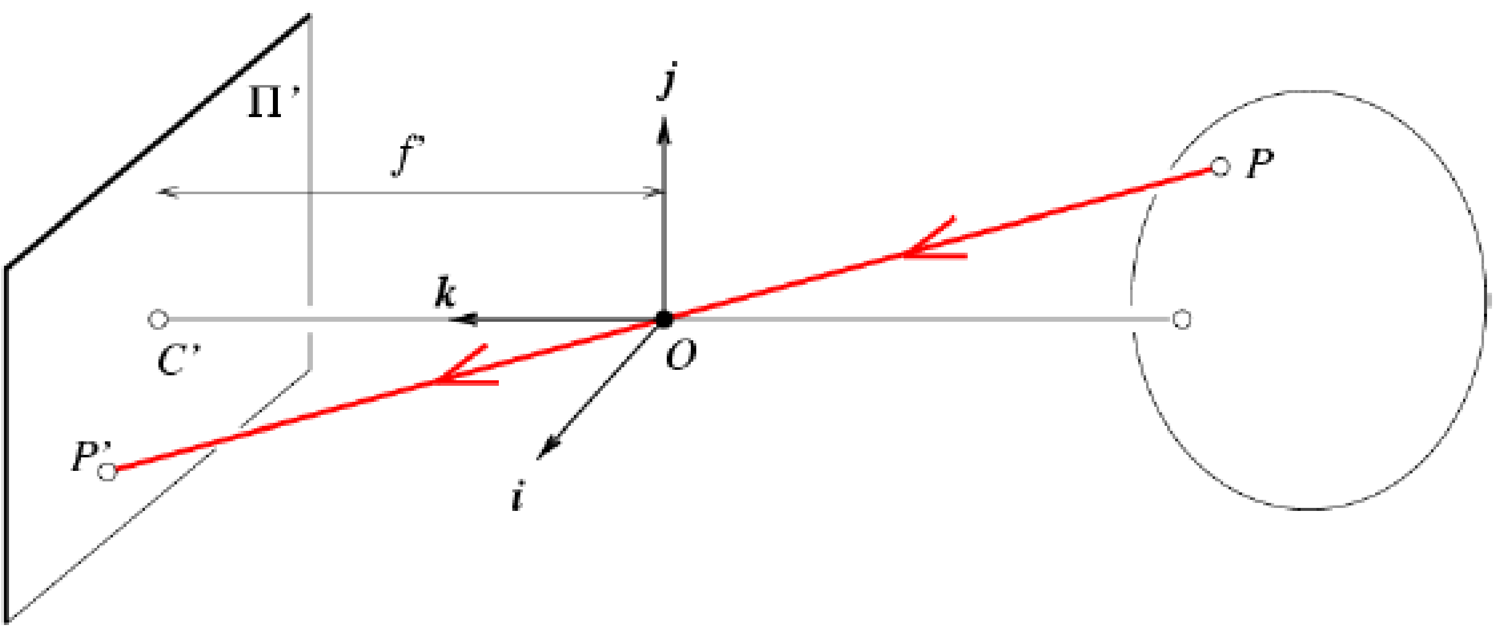


Camera Intrinsics

$$\tilde{\boldsymbol{x}}_s = \overbrace{\begin{bmatrix} f_x & s & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\boldsymbol{K}} \hspace{-4pt}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\;\; \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

- Optical center at origin
- ~~No skew~~
- ~~Unit aspect ratio~~

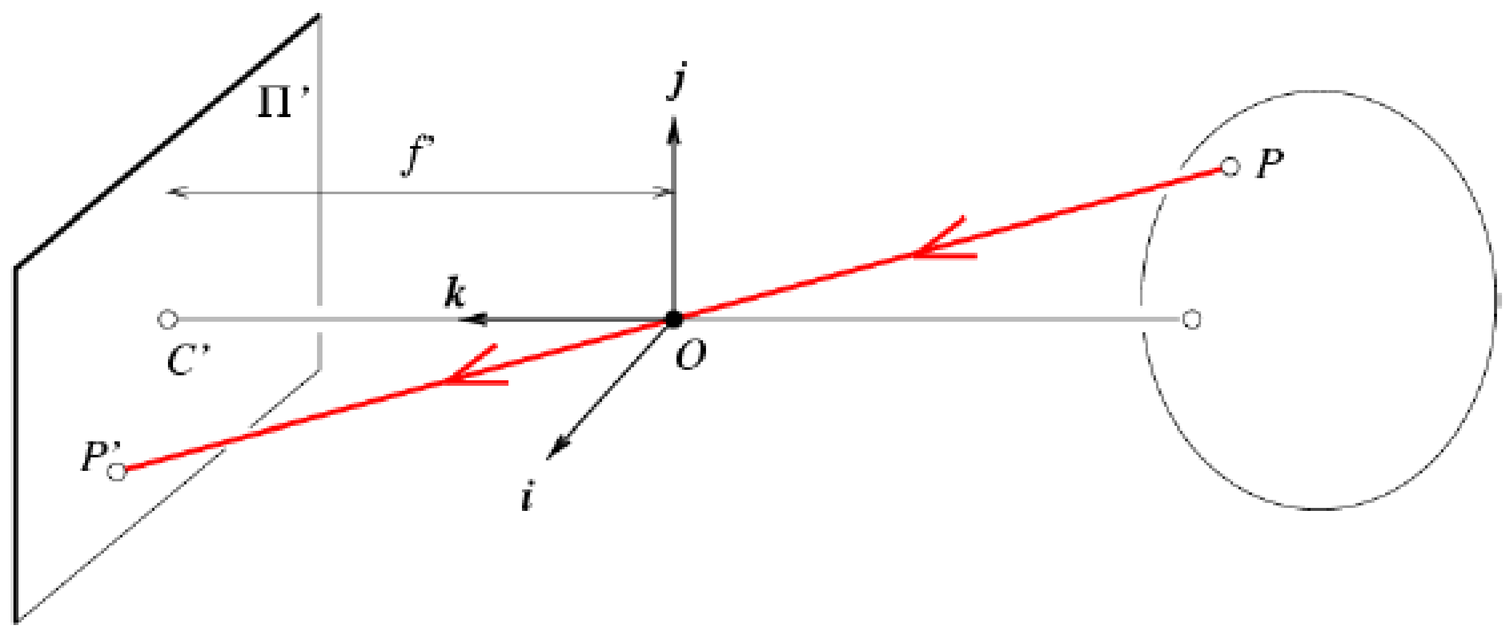


Camera Intrinsics

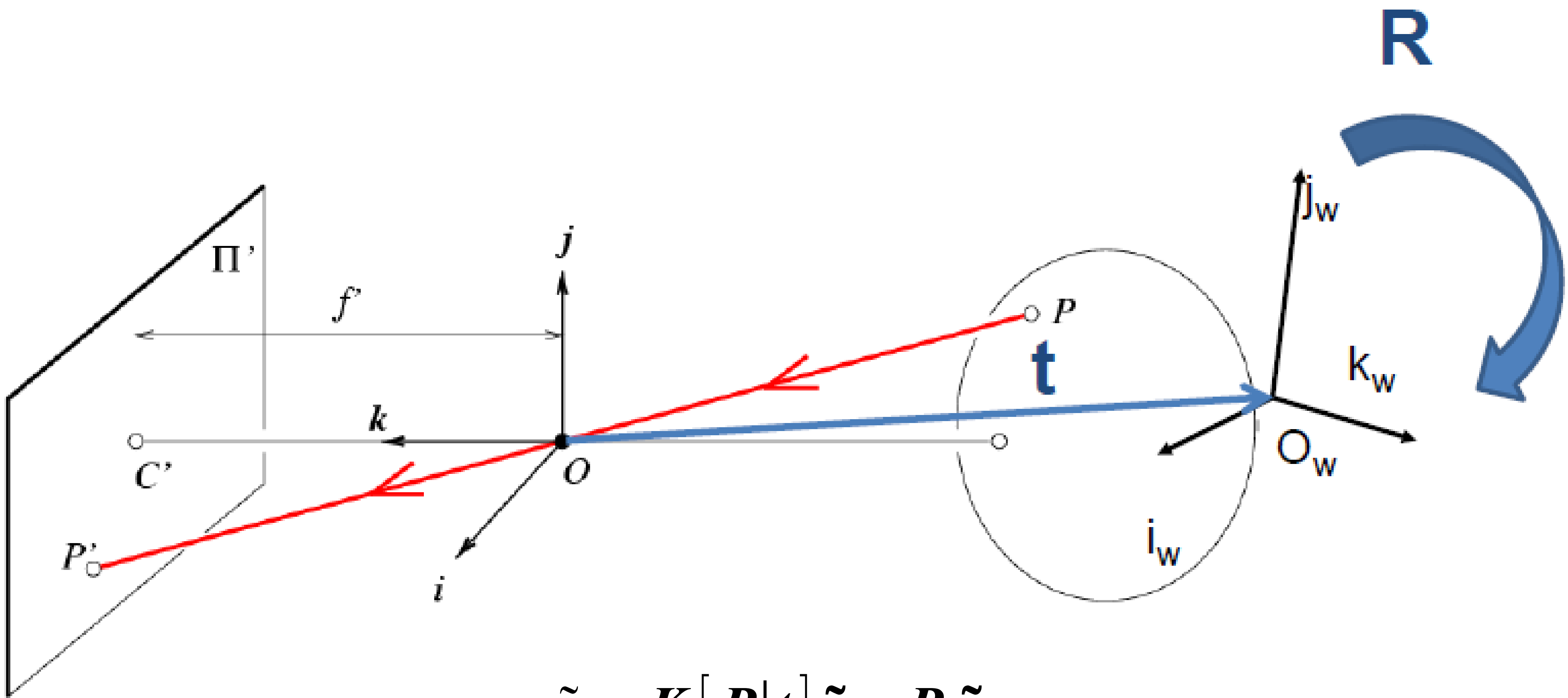
$$\tilde{\boldsymbol{x}}_s = \underbrace{\begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{K}} \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

- ~~Optical center at origin~~
- ~~No skew~~
- ~~Unit aspect ratio~~



Camera Extrinsics



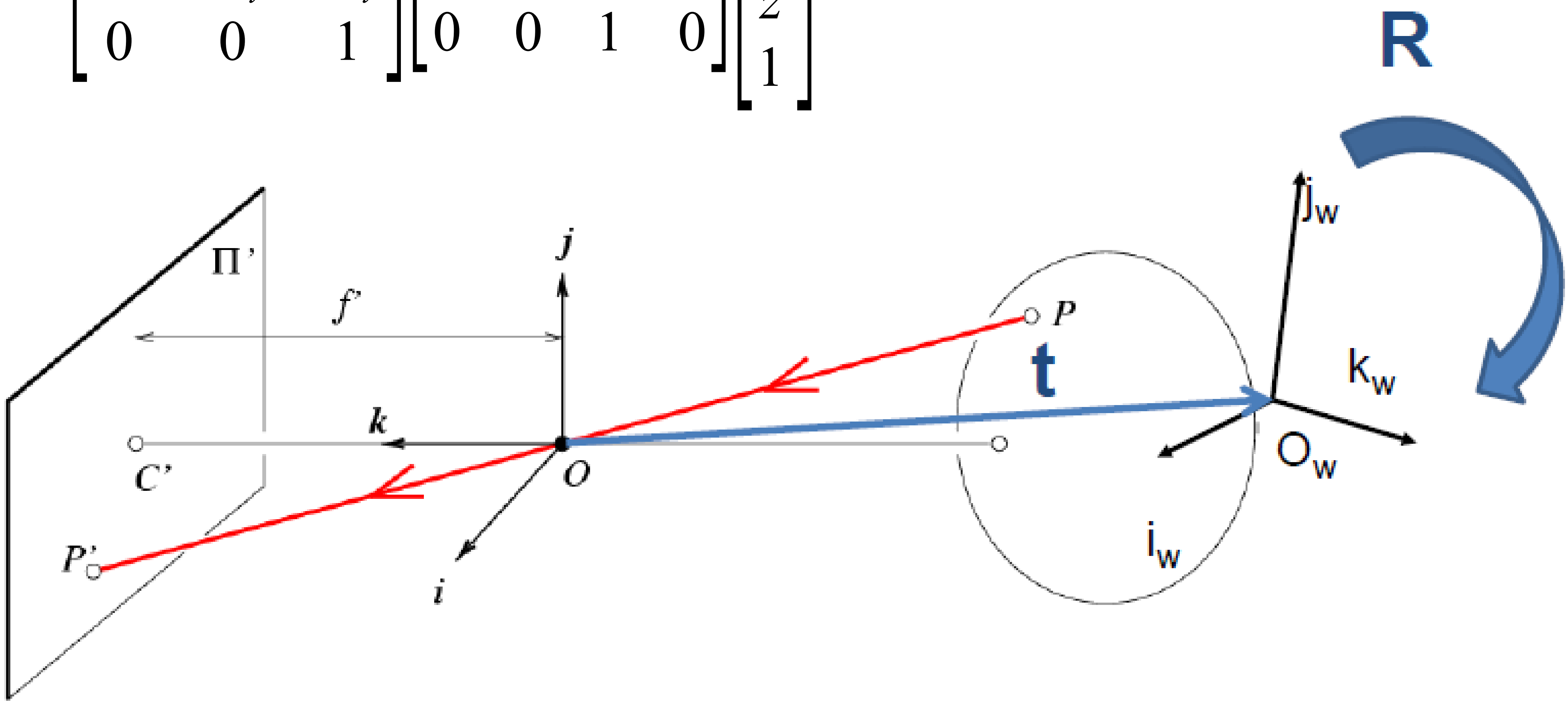
$$\tilde{x}_s = K\left[R|t\right]\tilde{p} = P\,\tilde{p}$$

Camera Extrinsics

$$\tilde{\boldsymbol{x}}_s = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

- No Translation
- No Rotation

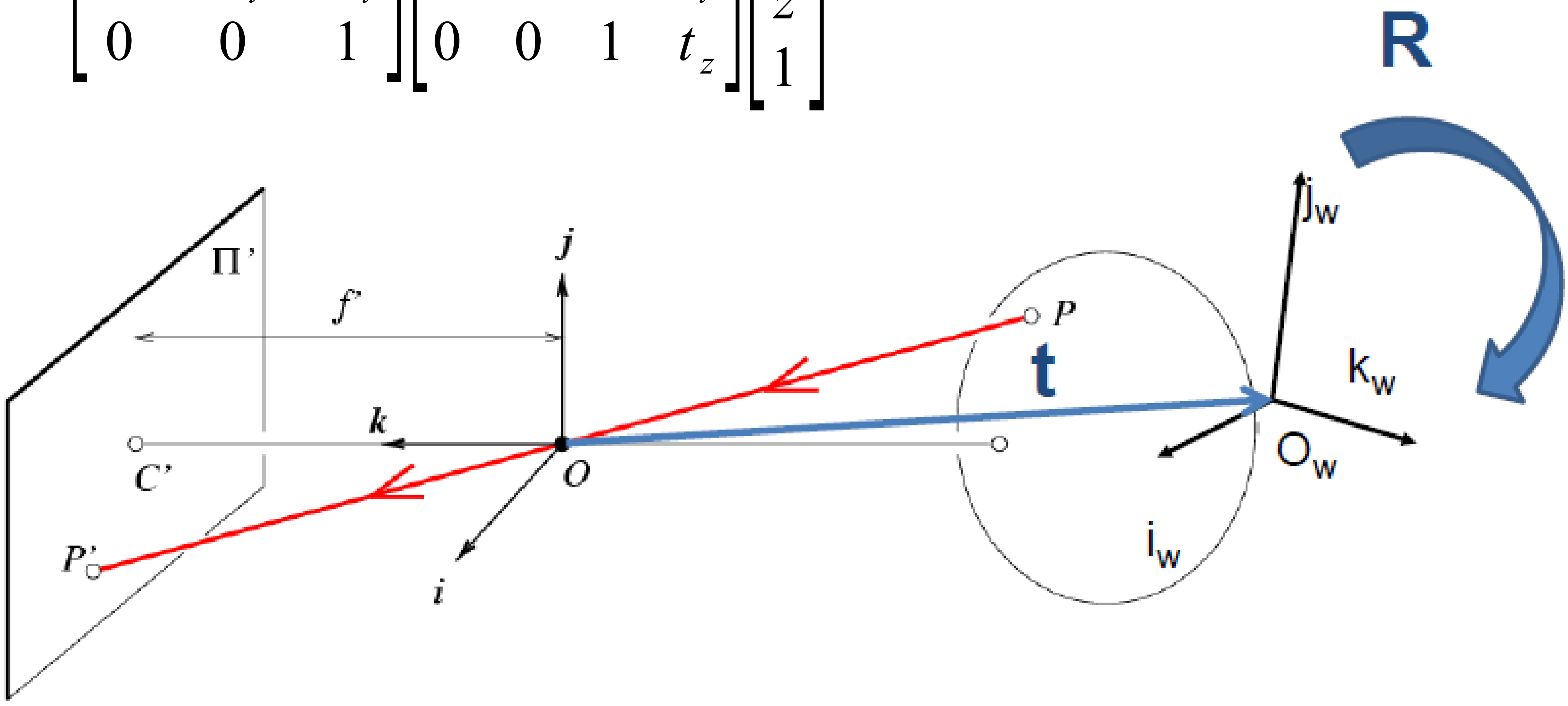


Camera Extrinsics

$$\tilde{\boldsymbol{x}}_s = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

- ~~No Translation~~
- No Rotation



Camera Extrinsics

$$\text{rotate-z}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rotate-x}(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

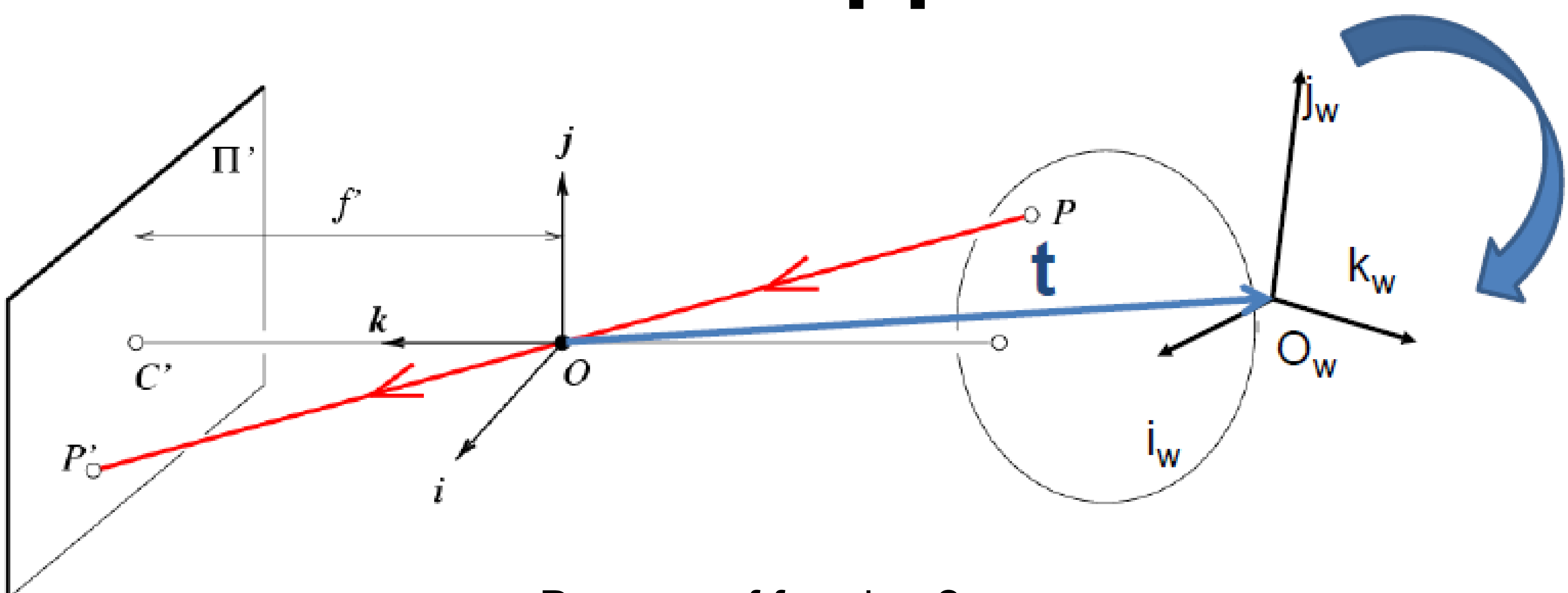
$$\text{rotate-y}(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

Camera Extrinsics

$$\tilde{\boldsymbol{x}}_s = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Assumptions:

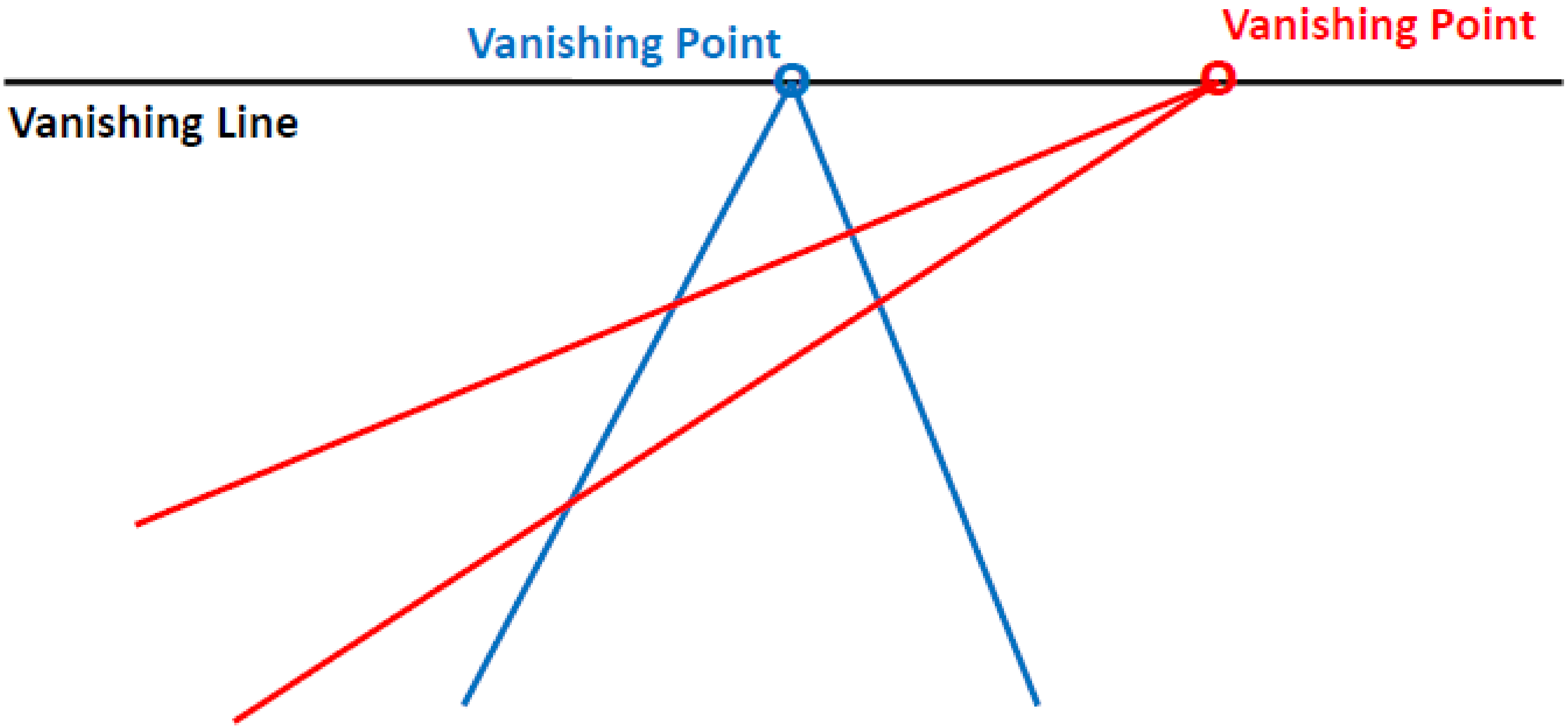
- ~~No Translation~~
- ~~No Rotation~~



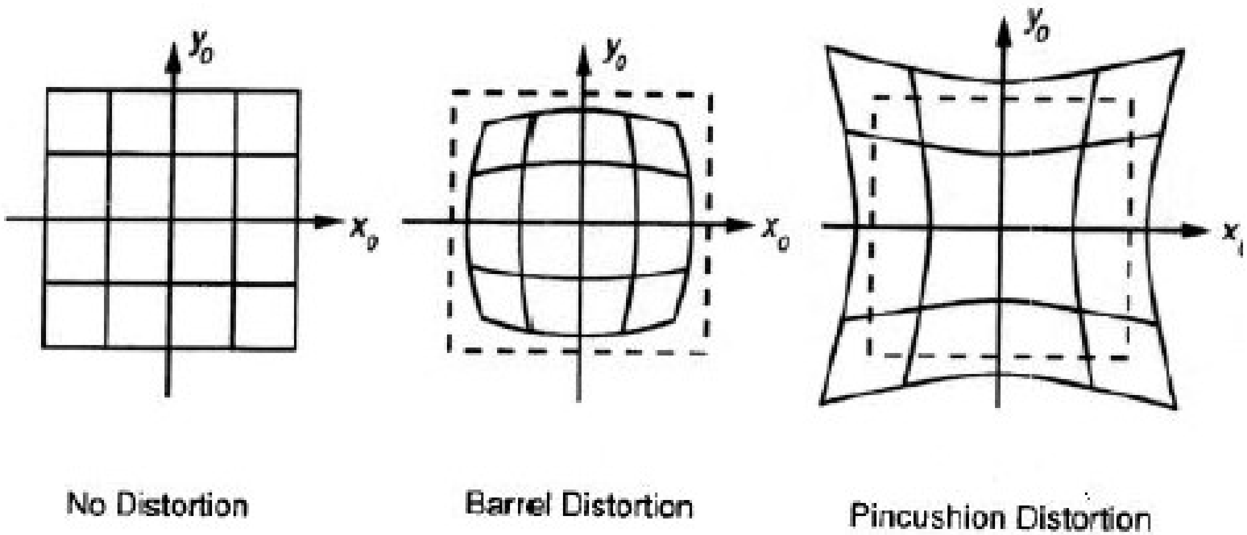
Degrees of freedom?

Vanishing Points

Vanishing Points



Radial Distortion



Corrected Barrel Distortion

Recap

- Pinhole Camera Model
- 2D and 3D Transformations
- Camera Matrix
- Vanishing Points
- Radial Distortion